



## Effects of thermophoresis and mixed convection on Carreau fluid flow with gold nanoparticles

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### Abstract

This study examines non-Newtonian nanofluid flow with heat transfer via a non-Darcy porous medium in the presence of effects. Additionally, the effects of the heat source, viscous and Ohmic dissipation, chemical reaction, electromagnetic field, and Biot number are taken into account. The non-linear equations that control the flow can be made simpler by using appropriate similarity transformations. Then, using a shooting and matching approach, the Rung-Kutta-Merson method is utilised to derive numerical solutions for the velocity, temperature, and concentration of nanoparticles as functions of the problem's physical parameters. This analysis could lead to the development of a model that could help in the understanding of the mechanics of physiological fluids. Additionally, the implications of these parameters on these solutions are studied numerically and graphically. It is discovered that as the Biot number rises or falls, both tangential and normal velocities change. While the temperature increases or decreases as the pressure gradient and radiation parameters increase, the concentration of nanoparticles increases or decreases as the Schmidt numbers and magnetic field parameters increase.

**Keywords:** Carreau nanofluid; heat transfer; thermophoresis effect; mixed convection; electromagnetic field; chemical reaction.

### 1. Introduction

Mixed convection takes place when the mechanisms of natural convection and forced convection work together to transfer heat. The phenomenon of mixed convection exists in several technical and industrial issues such as electronic devices chilled by fans, nuclear reactors refrigerated during the emergency stop, a heat exchanger placed in low-velocity surroundings, solar collectors, and so on. [1]. Mixed convective flows driven by differences in temperature and concentration at the different surface geometries through a porous medium have been extensively studied in the past and various extensions of the

problems have been reported in the literature. The Mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink is studied by Hayat et al. [2]. Pal and Mondal [3] investigated the effects of Soret and Dufour on MHD non-Darcian mixed convection heat and mass transfer over a stretching sheet with a non-uniform heat source/sink. Mixed convection in a vertical micro-annulus between two concentric micro-tubes was examined by Avcı and Aydın [4]. It became apparent that the increase in the mixed convection parameter increases the heat transfer.

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Some important studies on the subject can be found in [5–7].

Past two decades, non-Newtonian fluid flow studies were taking up greater attention and importance than Newtonian fluids, as far as their various industrial technical applications [8-14]. Nanofluids are a relatively new class of fluids that consist of a base fluid with nano-sized particles (1–50 nm) suspended within them [15]. Nanoparticles being used in nanofluids are usually made of metals, oxides, carbides, or carbon nanotubes. Nanofluids has numerous applications such as transportation (engine cooling/vehicle thermal management), electronics refrigeration, nuclear systems refrigeration, heat exchanger, chemical process, as well as biological medicine. Non-Newtonian nanofluids are commonly available in several industrial technology applications, for example, the thawing of polymers, biological solutions, paints, tars, asphalt, glues, etc. A notable example of these efforts is found in references [16-18]. The problem of MHD non-Newtonian nanofluid flow with a heat transfer under the effect of chemical reaction and radiation through a porous medium has been discussed by Eldabe et al. [19]. Various authors have explored the properties specific to different non-Newtonian nanofluid flows [20–29].

The main objective of this paper is to extend the work of Eldabe et al. [30] in the case of mixed convection, non-Darcian effect, and non-Newtonian nanofluid, viscous dissipation effect. Because the partial differential equations of velocity, temperature, and nanoparticle concentration are very highly non-linear, it has been transformed into a non-linear ordinary differential equation using the appropriate similarity transformations. This system of equations is solved numerically by the applying Rung-Kutta-Merson method with a Newton iteration in a shooting and matching technique. This analysis could make it a model that can support understanding the mechanics of physiological flows.

## 2. Mathematical formulations

Cartesian coordinates ( $x, y, z$ ) are taken into account, where  $x$  is along a direction of fluid flow,  $y$  is the normal to  $x$ , and  $z$  perpendicular plane ( $x, y$ ). The electrically conducting Carreau nanofluid is flowing

steadily into a shrinking sheet. The external applied magnetic field, while the electric field.

A constitutive model of Carreau fluid may be written as follows:

$$\tau_{ij} = -\eta_0 \left[ 1 + \frac{(n-1)}{2} \Gamma^2 \dot{\gamma}^2 \right] \dot{\gamma}_{ij}, \quad (1)$$

$$\dot{\gamma}_{11} = 2 \frac{\partial u}{\partial x}, \quad \dot{\gamma}_{12} = \dot{\gamma}_{21} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \dot{\gamma}_{22} = 2 \frac{\partial v}{\partial y}, \quad (2)$$

where  $\dot{\gamma}$  is defined as:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \dot{\gamma}_{ij} \dot{\gamma}_{ij}} = \sqrt{\frac{1}{2} \Pi_{\dot{\gamma}}}, \quad (3)$$

where  $\Pi_{\dot{\gamma}}$  is a second invariant of strain-rate tensor  $\dot{\gamma}_{ij}$ .

The equations of continuity, momentum, energy, and nanoparticles concentration can be written, respectively, as [31-33]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (4)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-1}{\rho} \frac{\partial p}{\partial x} - \frac{\eta_0}{\rho} \frac{\partial}{\partial y} \left\{ \left[ 1 + \frac{n-1}{2} \Gamma^2 \left( \frac{\partial u}{\partial y} \right)^2 \right] \frac{\partial u}{\partial y} \right\} + \quad (5)$$

$$\frac{\sigma}{\rho} (E_0 B_0 - B_0^2 u) + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty) - \frac{v}{k} u + C^* u^2,$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{K}{\rho c_p} \frac{\partial^2 T}{\partial y^2} -$$

$$\frac{\eta_0}{\rho c_p} \left\{ \left[ 1 + \frac{n-1}{2} \Gamma^2 \left( \frac{\partial u}{\partial y} \right)^2 \right] \frac{\partial u}{\partial y} \right\} \frac{\partial u}{\partial y} + \quad (6)$$

$$\frac{\sigma}{\rho c_p} (u B_0 - E_0)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p} (T - T_\infty) +$$

$$D_T \left( \frac{\partial T}{\partial y} \right)^2 + D_B \left( \frac{\partial T}{\partial y} \right) \left( \frac{\partial C}{\partial y} \right),$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_0} \frac{\partial^2 T}{\partial y^2} - A(C - C_\infty), \quad (7)$$

where the thermal radiation heat flux;

$$q_r = -\frac{4\sigma^*}{3k_0} \frac{\partial T^4}{\partial y}$$

We expect that the differences in fluid-phase temperatures in flow are enough small that  $T^4$  can be expressed as the linear function of temperature.

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

Simplifying the foregoing system, we apply the following transformations:

$$\begin{aligned} f''^2 - ff'''' &= -\frac{\partial p}{\partial x} - \frac{1}{\text{Re}} \left(1 + \frac{n-1}{2} \text{We} f''^2\right) f'''' - \\ &\left(M^2 - \frac{1}{\text{Da}}\right) f' + M^2 E_1 + G_r \theta + B_r \phi + F_s f''^2, \\ f'\theta' &= \frac{1+4R}{3\text{Pr}} \theta'' - \frac{\text{Ec}}{\text{Re}} \left(1 + \frac{n-1}{2} \text{We}^2 f''^2\right) f''^2 + \\ &\text{Ec} M^2 (f' + E_1)^2 + Q_0 \theta + \text{Nt} \theta' + \text{Nb} \theta' \phi', \\ \text{Sc} f' \phi' &= \phi'' + \frac{\text{Nt}}{\text{Nb}} \theta'' - \delta \phi'''. \end{aligned} \quad (11)$$

It should be noted that  $n=1$  leads to a boundary-layer flow of ordinary Newtonian conducting fluid. At the

same time when we put  $F_s = \frac{\partial p}{\partial x} = 0$ ,  $n=1$  and  $\text{Bi} \rightarrow \infty$ ,

this problem was investigated for the same boundary conditions by Eldabe et al. [30]. Eqs. (10)-(12) are coupled non-linear ordinary differential equations of order three. For Carreau fluid, as the parameter  $n$  tends to zero, the fluid is becoming ordinary Newtonian.

The boundary conditions in the non-dimensional form are:

$$\begin{aligned} f(0) &= 0, \quad f'(0) = 1, \quad \theta(0) = 1 + \frac{1}{\text{Bi}} \theta'(0), \quad \phi(0) = 1 \\ f'(\infty) &= \theta(\infty) = \phi(\infty) = 0. \end{aligned} \quad (13)$$

### 3. Numerical solutions

NAG Fortran libraries with help of subroutine D02HAF will be used to solve the foregoing system of equations (10-12). Moreover, then, the shooting

technique is applied. The present subroutine might be needed to suppose missing initial and terminal conditions. The governing equations (10-12) have been solved by the Rung-Kutta-Merson method of order five. In the present subroutine, we have used variable step size to control a local truncation error, but then again, the modified Newton-Raphson method is used to get successive corrections at the estimated boundary values.

The process is repeated iteratively several times before convergence and accuracy have occurred.

### (14) Discussion

Here, we analysed graphically how the temperature, nanoparticle concentration, and the problem's physical parameters affected the tangential and normal velocities. Moreover, we obtained the numerical values of these physical quantities and tabulated the coefficients of skin-friction and both heat transfer and mass transfer by using Mathematica package Ver.10.1. The standard values of these parameters are taken as follows:

$$\begin{aligned} n &= 2, \quad M = 0.5, \quad \text{Da} = 0.1, \quad E_1 = 0.5, \quad \frac{\partial p}{\partial x} = -10, \\ \text{Re} &= 0.5, \quad F_s = 0.4, \quad \text{We} = 0.5, \quad \text{Gr} = 0.5, \quad \text{Br} = 0.5, \quad \text{Pr} = 1, \quad R = 1 \\ \text{Ec} &= 3.5, \quad Q_0 = 1, \quad \text{Sc} = 2.5, \quad \text{Bi} = 0.5, \quad \text{Nt} = 3.5, \quad \text{Nb} = 2.5, \\ m &= 2, \quad \delta = 0.8. \end{aligned}$$

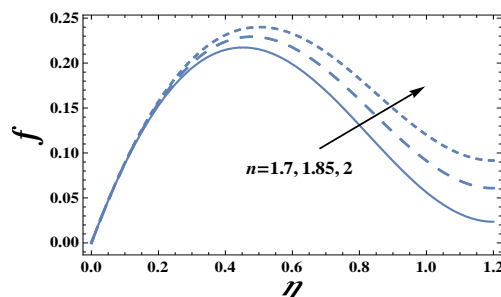


Fig. (1). The variation of  $f$  with  $\eta$ , for different values  $n$ .

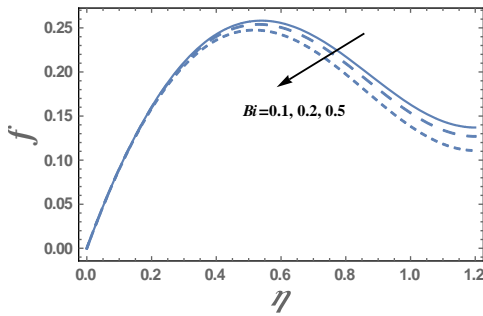


Fig. (2). The variation of  $f$  with  $\eta$ , for different values  $Bi$ .

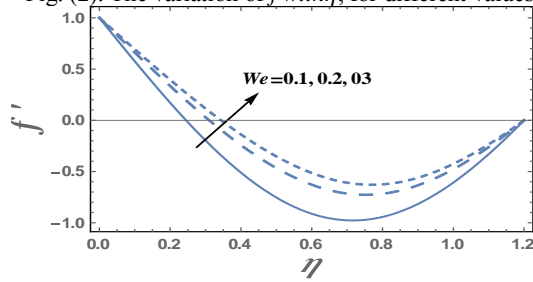


Fig. (3). The variation of  $f'$  with  $\eta$ , for different values  $We$ .

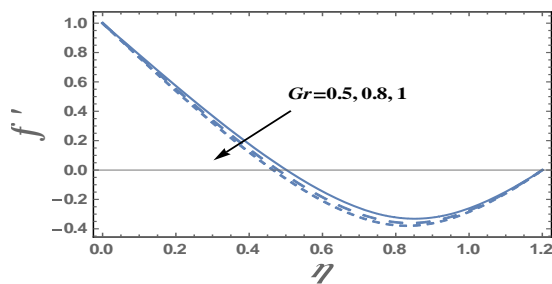


Fig. (4). The variation of  $f'$  with  $\eta$ , for different values  $Gr$ .

Biot number is a dimensionless quantity that measures the ratio of the reluctance of heat transfer from the inside of the body to the surface of the body. So, a small Biot number reveals low resistance to transport by conduction, and therefore very low-temperature gradients within the body. Figures (1) and (2) display the variations of the normal velocity  $f$  versus the dimensionless coordinate  $\eta$  for different values of dimensionless power-law index  $n$  and Biot number  $Bi$ , respectively. It is noted from these figures that the normal velocity increases with the increase of  $n$ , while it decreases as  $Bi$  increases. The result in Fig. (2) is due to the above definition of the Biot number. In addition,  $f$  increases with  $\eta$  for large

values of  $n$ , and small values of  $Bi$ , till a definite value  $\eta = \eta_0$  (represents the maximum value of  $f$ ) and it decreases afterward. Furthermore, all curves for different values of both  $n$  and  $Bi$  are identical near the sheet, namely in the interval  $\eta \in [0, 0.21]$ , otherwise, the effect of these parameters is clear; this is due to the fact that the shrinking sheet is cold initially. The Weissenberg number is defined by the ratio between elastic and viscous forces. Moreover, it usually measures the relation between the fluid stress relaxation time and a specific process time, i.e. Weissenberg number may help to increase the fluid flow. The variations of the tangential velocity  $f'$  with the dimensionless coordinate  $\eta$  for various values of Weissenberg number  $We$  and Thermal Grashof number  $Gr$  are shown in Figs. (3) and (4), respectively, The graphical results of Figs. (3) and (4), indicate that the tangential velocity increases with an increase in the parameter  $We$ , while it decreases by increasing the parameter  $Gr$ . Furthermore, It is observed that for small values of  $We$  and large values of  $Gr$ , the relation between  $f'$  and  $\eta$  is a parabola with a down vertex, i.e.  $f'$  decreases with  $\eta$  till a definite value  $\eta = \eta_0$ , (represents the minimum value of  $f'$ ) and it increases afterward. This absolute minimum value of  $f'$  increases by increasing  $Gr$ , while it decreases by increasing  $We$ . The above definition of the Weissenberg number explains the result in Fig. (3). Moreover, this result agrees with those presented by [30].

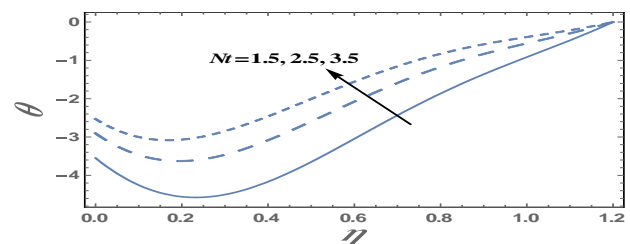


Fig. (5). The variation of  $\theta$  with  $\eta$ , for different values  $Nt$ .

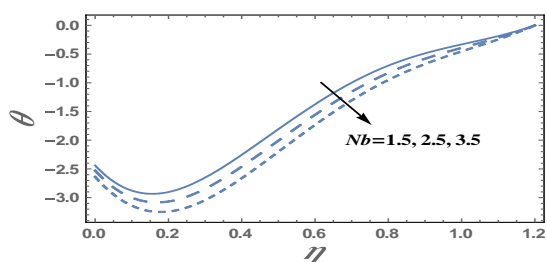


Fig. (6). The variation of  $\theta$  with  $\eta$ , for different values  $Nb$ .

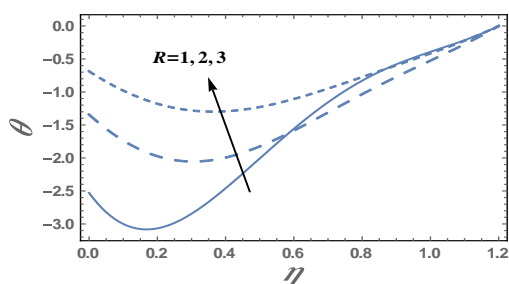


Fig. (7). The variation of  $\theta$  with  $\eta$ , for different values  $R$ .

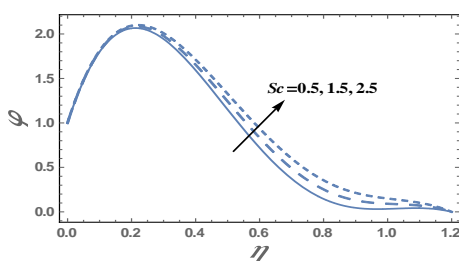


Fig. (8). The variation of  $\phi$  with  $\eta$ , for different values  $Sc$ .

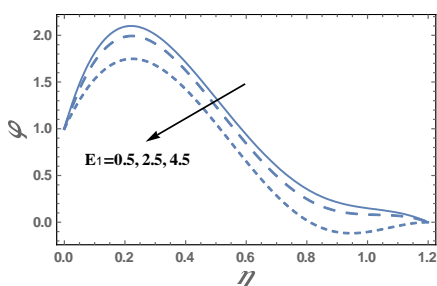


Fig. (9). The variation of  $\phi$  with  $\eta$ , for different values  $E_1$ .

Figs. (5) and (6) show the behavior of the temperature distribution  $\theta$  with the dimensionless coordinate  $\eta$  for various values of the thermophoresis parameter  $Nt$  and Brownian motion parameter  $Nb$ , respectively. It has been seen from these figures that the temperature increases with the increase of  $Nt$ , while it decreases as  $Nb$  increases. It is also noted

that for each value of both  $Nt$  and  $Nb$ , there exists a minimum value of  $\theta$  whose absolute value increases by increasing  $Nb$  and decreases by increasing  $Nt$ , and all minimum values occur at  $\eta=0.17$ . Brownian motion is an inherent flow of particles dangled in a fluid. This random transport agrees with the fact that the temperature decreases with the Brownian motion parameter. So, the result in Fig. (6) agrees with the physical expectation, and is in agreement with those which are presented by [26]. The effect of radiation parameter  $R$  on the temperature distribution  $\theta$  as a function of the dimensionless coordinate  $\eta$  is shown in Fig. (7). It is found that the temperature distribution increases by increasing  $R$  in the interval  $\eta \in [0, 0.65]$ ; otherwise it decreases by increasing  $R$ . So, the behavior of  $\theta$  in this interval is an inversed manner of its behavior in the interval  $\eta \in [0.65, 0.18]$ , except that the curves are quite close to each other in the second interval.

Eq. (12) evaluates how the nanoparticles concentration distribution  $\phi$  variations with the dimensionless coordinate  $\eta$ . The effects of both Schmidt number  $Sc$  and the local electric parameter  $E_1$  on the nanoparticles concentration distribution  $\phi$  are given in figures (8) and (9), respectively. It is found that the nanoparticles concentration increases by increasing  $Sc$ , but it decreases by increasing  $E_1$ . Furthermore, the nanoparticles concentration is always positive, and for large values of  $Sc$  and small values of  $E_1$ , it increases with  $\eta$  till a maximum value of  $\eta$ , after which it decreases.

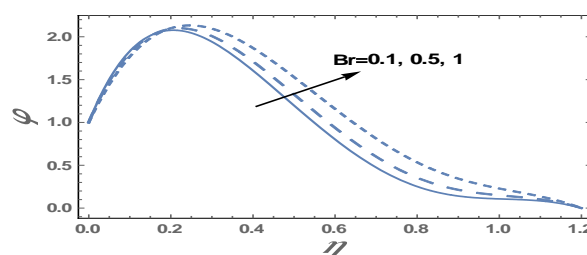


Fig. (10). The variation of  $\phi$  with  $\eta$ , for different values  $Br$ .

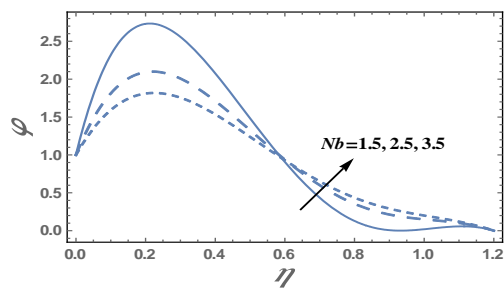


Fig. (11). The variation of  $\phi$  with  $\eta$ , for different values  $Nb$ .

The effect of nanoparticles concentration Grashof number  $Br$  on the nanoparticles concentration  $\phi$  which is a function of  $\eta$  is given in Fig. (10). It is found that the nanoparticles concentration decreases by increasing  $Nb$  in the interval  $\eta \in [0, 0.21]$ ; otherwise it increases by increasing  $\eta$ . So, the behavior of  $\phi$  in the interval  $\eta \in [0, 0.21]$ , is an inversed manner of its behavior in the interval  $\eta \in [0.21, 1.2]$ , and in the first interval, there is a maximum value of  $\phi$  holds at  $\eta = 0.2$ . Figure (11) illustrates the effect of Brownian motion parameter  $Nb$  on the nanoparticles concentration  $\phi$  as a function of  $\eta$ . It is found that the behavior of  $\phi$  for various values of  $Nb$  is in the same manner as the behavior of  $\phi$  for various values of  $Br$  given in Figure (10), except that all curves are very ruffled than to each other than those obtained in Fig. (10).

## 5. Conclusion

This problem extends the work of Eldabe et al. [30] to include mixed convection, non-Darcian effect, and non-Newtonian nanofluid, viscous dissipation effect. Since the partial differential equations of velocity, temperature, and nanoparticles concentration are very highly non-linear, it has been converted into the non-linear ordinary differential equation by using suitable similarity transformations. This system of equations is solved numerically by applying the Rung-Kutta-Merson method with a Newton iteration in a shooting and matching

technique. This analysis can render a model which may support comprehension of the mechanics of physiological flows [34-50]. The obtained results can be outlined as follows.

- 1) By increasing  $n$  and  $Sc$ ,  $n$ ,  $Br$ , and  $\delta$  both the normal and tangential velocities increase while they decrease as  $E_1$ ,  $M$ , and  $Bi$  increase. Moreover, they increase or decrease as  $Da$  increases.
- 2) The normal velocity becomes greater with increasing the dimensionless coordinate  $\eta$  and reaches a maximum after which, it decreases, but the tangential velocity has an opposite manner, i.e. it has a minimum value.
- 3) The temperature distribution increases as  $Da$ ,  $\delta$ ,  $Nt$ ,  $We$ , and  $Re$  increase, while it decreases or increases as  $Q$  and  $Nb$  increase. Furthermore, it decreases or increases as  $Bi$ ,  $Ec$ ,  $Gr$ ,  $Pr$ ,  $R$ ,  $Br$ , and  $Da$  increase.
- 4) The temperature becomes lower with increasing the dimensionless coordinate  $\eta$  and reaches a minimum at  $\eta = 0.18$ , after which, it increases.
- 5) The nanoparticles concentration has an opposite behavior with respect to the temperature behavior except that it decreases with the increase of  $\delta$ ,  $Gr$ ,  $Br$ , and  $We$ .

## Data availability

The datasets generated and/or analyzed during the current study are not publicly available due to [All the required data are only with the corresponding author] but are available from the corresponding author on reasonable request.

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**Conflict of interest**

The authors have declared that this paper has no conflict of interest.

**Nomenclature**

$A$	Reaction rate constant
$B_0$	Constant
$Bi$	Biot number
$Br$	Concentration Grashof number
$C$	The nanoparticles concentration
$C^*$	Forchheimer $\delta$ constant
$cp$	The specific heat at constant pressure
$Da$	Darcy number
$D_B$	Brownian diffusion coefficient
$D_T$	Thermophoretic diffusion coefficient
$E_0$	Constant
$E_1$	Local electric parameter
$Ec$	Eckert number
$Fs$	Forchheimer number
$p$	The fluid pressure
$Pr$	Prandtl number
$q_r$	Thermal radiation heat flux
$Q_0$	Heat source parameter
$R$	Radiation parameter
$Re$	Reynolds number
$Sc$	Schmidt number
$T$	The fluid temperature
$u$	The tangential component of velocity
$v$	The normal component of velocity
$We$	Weissenberg number
$x$	Tangential coordinate
$y$	Normal coordinate

**Greek symbols**

$\delta$	Chemical reaction parameter
$\eta_0$	Zero-shear-rate viscosity
$\Gamma$	time constant
$\nu$	Kinematic viscosity
$\rho$	Fluid density

$\sigma$	The electrical conductivity of the fluid
$\sigma^*$	Stefan-Boltzmann constant
$\tau_{ij}$	the stress tensor components,

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